EXECUTIVE SUMMARIES

2025

An Executive Summary consists of a brief description of the problem followed by a largely equation-free summary of the progress made and the results obtained by the study group.

REVIVING THE ECOSYSTEM IN BUSHBUCKRIDGE NATURE RESERVE

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Executive Summary

Bushbuckridge Nature Reserve (BNR), located in Mpumalanga, South Africa, is a protected area with significant ecological and socio-economic importance. Once a thriving ecosystem, the reserve has faced challenges such as habitat degradation, increased human settlements, and unsustainable resource over- utilisation, leading to biodiversity loss and escalating human-wildlife conflicts. Co-managed by the Mpumalanga Tourism and Parks Agency (MTPA) and the local Communal Property Associations (CPAs), BNR aims to restore its natural habitat while promoting sustainable resource use by surrounding communities. Strategically positioned as an ecological corridor between the Kruger National Park and the Blyde River Canyon Nature Reserve, BNR plays a crucial role in regional conservation efforts. Through planned restoration activities, scientific research, and community engagement, the reserve seeks to balance conservation with economic benefits, reinforcing its potential as a key player in South Africa's wildlife tourism industry.

The primary objective of the study group was to develop a mathematical model for the revitalisation of Bushbuckridge Nature Reserve to restore biodiversity, address human-wildlife conflict, and optimise resource allocation. This was carried out by presenting two models

1. Vegetation and species growth model, and

2. Facility layout design model,

which integrated ecological, environmental, and socio-economic factors to ensure effective revitalisation and sustainable conservation efforts in BNR while benefiting surrounding communities.

We first developed a model based on the Lotka-Volterra equations, capturing interactions between vegetation, prey, and predators. The model incorporates controlled human interventions such as reforestation, species reintroduction, and regulated harvesting (cutting of plants by surrounding communities) as well as regulated hunting by the surrounding community. The model was extended to a reaction-diffusion model to simulate vegetation growth and species movement over space and time, demonstrating the long-term viability of the reserve's restoration efforts. An optimisation-based space allocation model then was formulated to strategically distribute wildlife and infrastructure within the reserve. This ensures that herbivores and vegetation resources are positioned near park boundaries for easy community access, thereby reducing illegal activities and human-wildlife conflicts, while predators are placed centrally to maintain ecological stability. The simulation results indicate that the reserve can achieve a stable ecological balance where prey and predators coexist with regulated vegetation availability.

Key findings from this study group suggest that strategic planning and mathematical modelling are essential for ensuring the long-term success of BNRs restoration efforts. The study emphasises the importance of collaboration among stakeholders, including park management, local communities, and scientific institutions, to facilitate effective conservation practices. Future work will focus on integrating real-world data into the developed models and refining facility placement using non-linear optimisation techniques. These efforts aim to establish BNR as a sustainable nature reserve that supports biodiversity conservation and contributes to South Africas tourism sector.

MOSQUITO-BORNE DISEASE CONTROL

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Executive Summary

Mosquito-borne diseases pose significant public health challenges, particularly in regions such as Sub-Saharan Africa, South America, Southeast Asia, and the Western Pacific. Annually, these diseases contribute to millions of infections and thousands of fatalities, with Africa being disproportionately affected due to factors such as poverty, limited access to healthcare, and low adherence to preventive measures. These diseases are transmitted to humans through the bites of female mosquitoes that carry pathogens, which they seek out to obtain the blood necessary for reproduction. Notable examples of mosquito-borne diseases include Zika, Chikungunya, Dengue, Leishmaniasis, and Malaria.

Preventing mosquito-borne diseases primarily focuses on minimizing mosquito bites. In various countries, insecticide-treated bed nets (ITNs) have been extensively distributed. These nets serve as a physical barrier, protecting individuals from mosquito bites during sleep while simultaneously killing or repelling mosquitoes through the insecticide-infused fabric. However, the widespread use of ITNs may have unintended consequences on immunity development. Regular exposure to pathogens, such as the Plasmodium parasites responsible for malaria, is crucial for building immunity, particularly in endemic regions. By lowering infection rates, ITNs can hinder this immunity development, making individuals more vulnerable to severe mosquito-borne diseases when they eventually encounter these pathogens.

This project aimed to investigate the long-term effects of ITN usage on mosquito-borne disease transmission and immunity development through mathematical modeling. Our innovative model encompasses the following key components:

- The transmission dynamics of mosquito-borne diseases between humans and mosquitoes.
- The role of ITNs in reducing mosquito biting rates and increasing mosquito mortality.
- The lifecycle of mosquitoes, including their development from eggs and larvae to pupae, breeding sites, and their pursuit of blood.
- The process of immunity development informed by disease incidence.

Through mathematical analysis, we established the model's well-posedness and calculated the basic reproduction number. Our numerical simulations demonstrated the effectiveness of bed nets in alleviating the disease burden, while also highlighting their detrimental impact on immunity development.

Due to time constraints, we were unable to conduct a full analysis of the model. Future steps will include:

- Performing an in-depth mathematical analysis to identify the model's equilibria and assess their stability, along with the bifurcation points that influence its dynamics.
- Supporting our theoretical framework with numerical simulations and fitting our model to real-world data from a country where these diseases are endemic.
- Evaluating the trade-offs between ITN usage and immunity development, ultimately formulating relevant recommendations.

SUGAR CENTRIFUGE IMAGE CLASSIFICATION

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Executive Summary

The Sugar Milling Research Institute has developed a camera-based system to optimize the water quantity added during the sugar centrifugation process. This system employs a camera mounted atop the centrifuge, focused on the screen inside the machine. A stroboscopic LED light is triggered to freeze frames, enabling clear observation of the sugar flow on the screen. The objective of the system is to analyze these images to assess sugar quality and generate automated control actions to enhance the centrifugation process.

During the process, massecuite - a dark, viscous mixture - enters the centrifuge basket and moves up the screen, leaving molasses behind while expelling clean sugar over the rim. Depending on the feed characteristics, finger-like patterns may form on the screen surface. If too many of these fingers reach the rim, non-sugar material is unnecessarily reprocessed, reducing efficiency. Conversely, if the sugar appears overly clean, it may indicate excessive water usage, leading to sugar losses in the form of dissolved molasses. Accurate image analysis is therefore critical for achieving a balance between these extremes.

The current control system uses Otsu thresholding to binarize images and calculate a "sugar ratio" based on the number of white pixels in a specific region near the basket rim. However, this method is highly sensitive to lighting conditions, which can lead to unreliable results, particularly in areas with poor illumination. To address these limitations, the study group explored alternative image segmentation techniques that are more resilient to varying lighting conditions.

One promising approach considered was the use of Convolutional Neural Networks (CNNs), specifically the U-Net architecture, which is highly effective in image segmentation tasks. Initially designed for medical imaging, U-Net's adaptability makes it appealing for analyzing sugar centrifuge images. However, training such a neural network requires a well-curated dataset, which was unavailable. While efforts were made to curate data from centrifuge video footage, the institute lacks the resources necessary to fully implement this solution.

Instead, the study group tried to improve the existing image analysis process using K-means clustering. The proposed method involves three stages: preprocessing, clustering, and sugar ratio computation. During preprocessing, the image is cropped to focus on the region of interest, excluding irrelevant areas like the hub and rim, and adaptive histogram equalization is applied to improve contrast and lighting. K-means clustering is then used to segment pixel intensities into distinct clusters, enabling more effective differentiation between light and dark areas. Finally, the sugar ratio is calculated based on the cluster distributions near the basket rim. This improved technique outperformed Otsu thresholding by producing more reliable segmentation results, even in challenging lighting conditions.

The K-means clustering approach shows significant promise for optimizing sugar centrifuge control. However, additional improvements are needed to further enhance its robustness and reliability. Comprehensive testing across larger datasets is essential to validate its performance and establish a strong correlation between the computed sugar ratio and sugar purity as determined by laboratory testing. By addressing these areas, the methodology can become a practical and effective solution for the sugar industry, leveraging available resources while paving the way for future advancements.

SURVIVAL OF THE LITTLE AFRICAN PENGUIN UNDER CLIMATE CHANGE CONDITIONS

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Problem Description

The Little African penguin (Spheniscus demersus) is native to Africa and lives in 24 islands between Namibia and Algoa Bay near Port Elizabeth. The population has declined by 95% since pre-industrial times, and estimates in 2024 indicate there are about 20,000 breeding pairs remaining. The species is on the critically endangered list and under Cape Nature Conservation protection. It has been estimated that the species could go extinct by 2035 due primarily to poor breeding habitat and availability of food.

The increased temperatures in the water and on land associated with climate change cause heat stress for the penguins and this affects breeding success. Fewer eggs are laid, fledglings do not survive or are abandoned. Additionally the environment is modified (storm surges, rising sea levels, predators, habitat changes) and fewer penguins survive to maturity. On top of all this are the usual anthropogenic pressures (commercial fishing, oil spills, habitat destruction, tourist issues). However researchers in the area suggest that the major survival threats come from commercial fishing and uncertainty of the food supply associated with climate change.

The primary and most important source of food for penguins is sardines and the massive sardine run that (normally) occurs yearly (from May through to July) provides food for larger fish, sharks, dolphins as well as for the penguins, and of course commercial fishing also affects the supply. Anchovies are are good substitute for the penguins (but their fat content is less than that of sardines), and also squid and other sea foods. The sardine run has been unreliable in recent years presumably because of climate change. In fact the sardine run 'did not happen' in 2006 and this resulted in a major reduction in penguin numbers in the Robbin Island colony in years following. Extensive studies have been carried out on the Robbin Island colony [2] and this data has been used to calibrate the models developed during the MISG.

Given the above information it was decided to focus attention on the food supply and climate change issues.

Executive Summary

A global predator-prey type modal was developed to investigate the effect of changes in the average sardine supply rate on penguin breeding numbers over periods of years. In these models the effect of climate change on food supply and breeding success are contained in a food supply rate term and (birth and death rate) parameters that need to be estimated using available data. As would be expected the solutions indicate extinction under severe food shortage conditions either due to climate change or excessive commercial fishing. Under 'sustainable' supply conditions an equilibrium state is eventually reached but with oscillations in sardine and penguin numbers before reaching this state. The amplitude of the oscillations is dependent on the initiating circumstances which represents a major threat; the penguin population could well go extinct before reaching an equilibrium state due to 'a catastrophic event', for example a poor sardine run.

A major concern is a mismatch in timing between the breading season and the sardine run and this could greatly affect breeding success. The breeding penguins require roughly double the food supply rate to feed the developing family. Historically the two events have matched up; evidently a coevolutionary development. We are investigating the effect of such a timing mismatch using an adapted model.

The above model does not take into account penguin maturity and demographic issues; such issues are 'hidden' (and not correctly accounted for) in fitted parameters. Penguins take several years to reach maturity and evidently 'the burden' born by the colony both in terms of reproduction and food needs will vary depending on history. For example, a good breeding season could result in an 'excess' of needy dependents, and this could be catastrophic under stressful circumstances. With this in mind 'a penguin life cycle under stress' model is under development.

The parameters used in the above models were chosen more to illustrate possible outcomes rather than represent 'reality'. Further work will be needed to establish realistic input for the models.

References

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MATHEMATICAL MODELLING OF COMBINED UV-CHLORINE TREATMENT TO DISINFECT WATER

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Executive summary

Waterborne diseases are a significant global challenge and the provision of safe drinking water is a fundamental public health requirement. Disinfection plays a crucial role in water treatment processes by eliminating pathogenic microorganisms that can cause severe illness. Due to their proven efficacy, ease of application, and relatively low cost Chlorine-based disinfectants, such as sodium hypochlorite (NaClO) and chlorine dioxide (ClO_2), are among the most widely used methods for ensuring microbial safety in drinking water systems. They are capable of inactivating a broad range of microorganisms, including bacteria, viruses, and protozoa. However, despite their extensive use, chlorine-based disinfectants face increasing scrutiny due to challenges including the increased microbial resistance, and the formation of potentially harmful disinfection by-products (DBPs) due to the reaction with dissolved organic matter.

To address these limitations, ultraviolet (UV) radiation has been explored as a complementary disinfection method. UV-C light disrupts microbial DNA by causing the formation of thymine dimers, which block replication and transcription, ultimately leading to cell death. The combination of UV radiation with chlorine-based disinfectants presents a promising strategy for improving disinfection efficacy. Additionally, the use of UV light may allow for lower chlorine dosages while maintaining or even improving disinfection performance, potentially reducing the formation of DBPs.

During the week we developed an ODE system to describe the evolution and interaction of chlorine, bacteria, organic matter and UV. UV exposure of chlorine leads to the formation of chlorine radicals, as well as reactive oxygen species, which were incorporated into the system as a single term. The ODE system involved a high number of unknown constants, which would be difficult to quantify with the limited data provided, consequently we defined a hierarchy of reduced systems which could be easily tested experimentally.

The first reduced system involved simply chlorine and bacteria and could be solved analytically. Comparison with the standard Chick-Watson model demonstrated that the standard model is incapable of tracking the chlorine concentration, however our model, although somewhat better still predicted a lower chlorine reduction rate than in experiments. This clarified the need to include dead bacteria, as organic matter, which still consumes chlorine.

Other systems were open to numerical solution. We also developed a small-time solution for the full system. Overall the results are meaningless without validation (or not) from experimental data. Consequently the recommendation was to carry out a series of further experiments, to test the models and isolate different effects. From this we should be able to estimate most of the unknowns before fitting to the data for the most complex experiments. Once all unknowns are determined we will be in a position to optimise or at least improve the combined chlorine-UV disinfection process.

INNOVATIVE ROOF SYSTEM FOR LOW-COST HOUSING

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Executive Summary

Housing for poorer sections of the South African population is a major issue. Our industry partners built a prototype low-cost house on the West Campus at the University of the Witwatersrand, using cement stabilized earth for the curved (double-vaulted) roofs. This structure was built with the goal to not only provide cheaper housing, but also to provide a more comfortable thermal environment for the occupants. This prototype was monitored over a number of years, with data for temperature in the northern and southern rooms and outside, including scenarios of open windows and closed windows throughout the year. The group was charged with doing some modeling to determine the effectiveness of this design and to estimate heat exchange through the roof and throughout the building, with a view to doing more complex modeling of air flow in the house, potentially identifying design improvements.

Several models for the thermal performance of the building and roof structure were considered, but the group finally settled on two different approaches. The first was to write a simple, vertical one- dimensional model that incorporated heating from the sun, radiative transfer and heat exchange through the roof, and the second was a finite element model that included a shaped roof, including both sides of the two-domed structure. Both models were used to make preliminary calculations of the heat in the structure for comparison with the data.

The one-dimensional model included a shape-factor function to take into account the different possible shapes of the roof structure, from flat to domed, and resulting computations of internal temperatures were consistent with the measurements. However, this model was limited to a single domed structure. The finite element model performed well in reproducing the heating phases of the situation for the double roof structure, but initially heat storage in the roof was not included, and so night time temperatures were sometimes underestimated.

However, the two models both produced promising results that can be improved by fine tuning. In the case of the one-dimensional model, incorporation of a second one-dimensional layer to represent the other side of the house should provide improved comparisons, while the inclusion of the heat sink of the roof in the finite element model is likely to provide much more accurate comparisons. The completion of this work should provide useful tools for further study of the effectiveness of the structure.

The project was successful in developing approaches that will provide useful information for the industry partners in future developments of the structure.